Electrical Technology

Contents

Series Resonance

The **peak power** delivered to the circuit is;

$$P = \frac{V_m^{\ 2}}{R}$$

The so-called half-power is given when $I = \frac{V_m}{\sqrt{2}R}$.

We find the frequencies, w_1 and w_2, at which this half-power occurs by using;

$$\sqrt{2}R = \sqrt{R^2 + (wL - \frac{1}{wC})^2}$$

Series Resonance

After some insightful algebra one will find two frequencies at which the previous equation is satisfied, they are:

$$w_1 = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

and

$$w_2 = \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

The two half-power frequencies are related to the resonant frequency by

$$w_o = \sqrt{w_1 w_2}$$

Series Resonance

The bandwidth of the series resonant circuit is given by;

$$BW = w_b = w_2 - w_1 = \frac{R}{L}$$

We define the Q (quality factor) of the circuit as;

$$Q = \frac{w_o L}{R} = \frac{1}{w_o RC} = \frac{1}{R}\sqrt{\left(\frac{L}{C}\right)}$$

Using Q, we can write the bandwidth as;

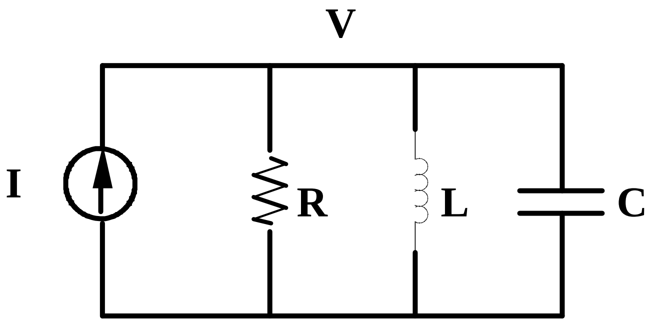
$$BW = \frac{w_o}{Q}$$

These are all important relationships.

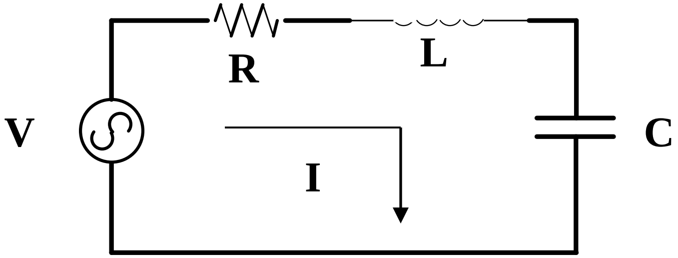
Parallel Resonance

Background

Consider the circuits shown below:



$$I = V\left[\frac{1}{R} + jwL + \frac{1}{jwC}\right]$$



$$V = I\left[R + jwL + \frac{1}{jwC}\right]$$

Series Resonance-Parallel Resonance

Duality

$$I = V\left[\frac{1}{R} + jwC + \frac{1}{jwL}\right] \qquad V = I\left[R + jwL + \frac{1}{jwC}\right]$$

We notice the above equations are the same provided:

$$I \longleftrightarrow V$$

$$R \longleftrightarrow \frac{1}{R}$$

$$L \longleftrightarrow C$$

If we make the inner-change, then one equation becomes the same as the other.

For such case, we say the one circuit is the dual of the other.

Parallel Resonance

Background

What this means is that for all the equations we have derived for the parallel resonant circuit, we can use for the series resonant circuit provided we make the substitutions:

$$R \quad \text{replaced be} \quad \frac{1}{R}$$

$$L \quad \text{replaced by} \quad C$$

$$C \quad \text{replaced by} \quad L$$

Series Resonance

Duality

$$I = V\left[\frac{1}{R} + jwC + \frac{1}{jwL}\right] \qquad V = I\left[R + jwL + \frac{1}{jwC}\right]$$

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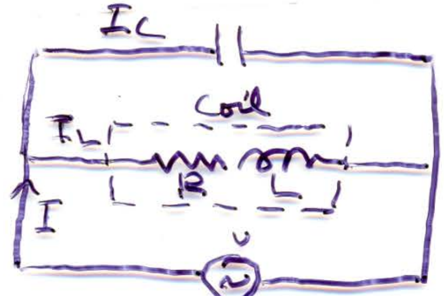
PARALLEL RESONANCE

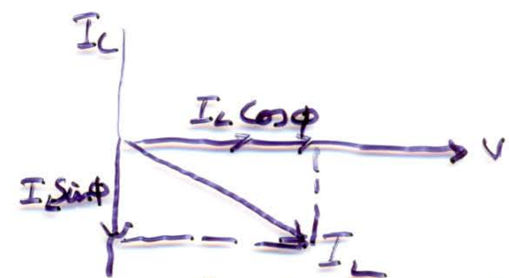
$$I_C = \frac{V}{X_C}$$

$$I_L = \frac{V}{Z_L} = \frac{V}{\sqrt{R^2 + X_L^2}}$$

$$\cos\phi = \frac{R}{Z_L}$$



Typical parallel Resonant Circuit



Vector Diagram of Parallel cct.

At resonance

$$I_C = I_L \sin\phi$$

(capacitive current = inductive part of coil current)

(ie imaginary components of I_L + I_C cancel each other

$$\frac{V}{X_C} = \frac{V}{Z_L} \times \frac{X_L}{Z_L}$$

$$Z_L^2 = X_C X_L$$

$$= \frac{1}{\omega_0 C} \times \omega_0 L = \frac{L}{C}$$

$$Z_L = \sqrt{\frac{L}{C}} \quad \text{———— (1)}$$

$$\sqrt{R^2 + \omega_0^2 L^2} = \sqrt{\frac{L}{C}}$$

$$R^2 + \omega_0^2 L^2 = \frac{L}{C}$$

$$\omega_0^2 L^2 = \frac{L}{C} - R^2$$

Q-factor

$$Q = \frac{I_C}{I} = \frac{V/X_C}{V/Z_r}$$

$$= \frac{Z_r}{X_C} = \frac{L}{CR} \times \frac{1}{\omega_r C}$$

$$= \frac{L}{CR} \times \omega_r C$$

$$= \omega_r \frac{L}{R}$$

$Q = \omega_r \frac{L}{R}$ = Q factor of series circuit

$$\omega_0^2 = \frac{1}{LC} - \frac{R^2}{L^2}$$

$$\omega_0 = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

If resistance of coil is neglected then

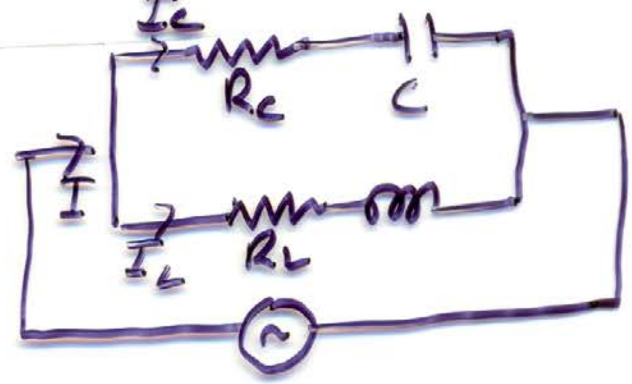
$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$ i.e. frequency of the resonance of the series cct.

At resonance, only $I_L \cos\phi$ remains

$$\frac{V}{Z_r} = \frac{V}{Z_L} \cdot \frac{R}{Z_L} \quad \text{or} \quad Z_r = \frac{Z_L^2}{R} = \frac{L}{CR} \quad \text{from (1)}$$

The equivalent impedance of the parallel resonating circuit is $\frac{L}{CR}$. This impedance is called DYNAMIC RESIS-TANCE of the parallel circuit. R is less, Impedance is very high at resonance. Current is much lower in parallel circuit. Thus this circuit is also called REJECTOR CIRCUIT. Pf is unity

RESONANCE BETWEEN PARALLEL R-C & R-L CIRCUIT



$$Y = \frac{1}{R_C - jX_C} + \frac{1}{R_L + jX_L}$$

$$= \frac{R_C}{R_C^2 + X_C^2} + \frac{R_L}{R_L^2 + X_L^2}$$

$$+ j\left[\frac{X_C}{R_C^2 + X_C^2} - \frac{X_L}{R_L^2 + X_L^2}\right] = G + jB$$

At resonance susceptance is zero i.e. $\frac{X_C}{R_C^2 + X_C^2} = \frac{X_L}{R_L^2 + X_L^2}$

If $R_C^2 = R_L^2 = \frac{L}{C}$ then

B is always zero.

This shows that the circuit will be at resonance for any frequency provided $R_C = R_L = \sqrt{\frac{L}{C}}$.

Again in this cct $I_c \sin\phi_1 = I_L \sin\phi_2$

$$\therefore \frac{V}{R_c^2 + \frac{1}{\omega_0^2 C^2}} \cdot \frac{\frac{1}{\omega_0 C}}{\sqrt{R_c^2 + \frac{1}{\omega_0^2 C^2}}} = \frac{V}{\sqrt{R_L^2 + \omega_0^2 L^2}} \cdot \frac{\omega_0 L}{\sqrt{R_L^2 + \omega_0^2 L^2}}$$

$$\left[\begin{array}{l} \sin\phi_1 = \frac{X_c}{Z_{Rc\,cct}} \\ \sin\phi_2 = \frac{X_L}{Z_{RL\,cct}} \end{array}\right.$$

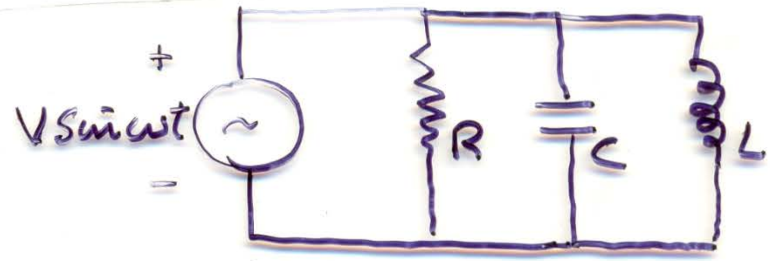
gives $\omega_0^2 \left[L^2 - R_c^2 LC\right] = \frac{L}{C} - R_L^2$

$$\omega_0^2 = \frac{\frac{L}{C} - R_L^2}{L^2 - R_c^2 LC}$$

$$\omega_0^2 = \frac{1}{LC}\left[\frac{\frac{L}{C} - R_L^2}{\frac{L}{C} - R_c^2}\right]$$

$$\omega_0 = \frac{1}{\sqrt{LC}} \sqrt{\frac{\frac{L}{C} - R_L^2}{\frac{L}{C} - R_c^2}}$$

PARALLEL RESONANCE



Parallel Resonance cct

$$Y(j\omega) = \frac{1}{R} + \frac{1}{j\omega L} + j\omega C$$

$$= G + j\left(\omega C - \frac{1}{\omega L}\right)$$

$$= G + jB \qquad (B = B_c - B_L)$$

$$|Y(j\omega)| = \sqrt{G^2 + B^2} = \sqrt{G^2 + \left(\omega C - \frac{1}{\omega L}\right)^2}$$

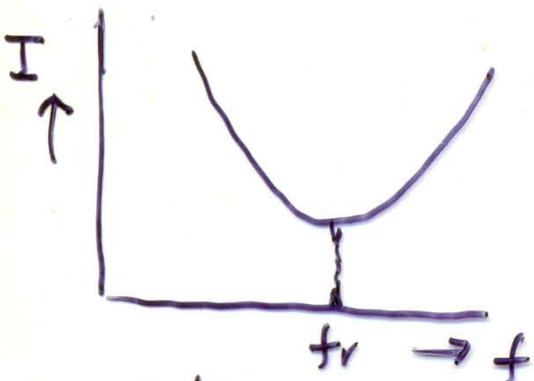
$$I = VY = V\sqrt{G^2 + \left(\omega C - \frac{1}{\omega L}\right)^2}$$

The cct is at resonance when Susceptance is Zero
The admittance then becomes Minimum. & = G.
The current the cct is minimum. $I = VG$
frequency is fr.

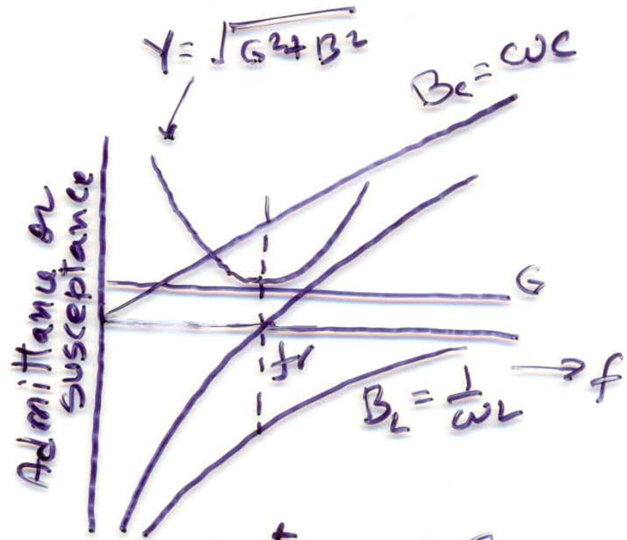
$$\omega_r C = \frac{1}{\omega_r L}$$

$$\omega_r^2 = \frac{1}{LC} \quad \text{or} \quad \omega_r = \frac{1}{\sqrt{LC}}$$

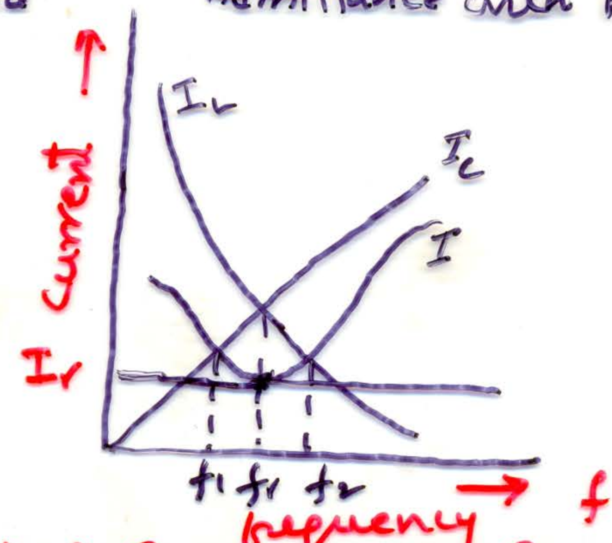
$$f_r = \frac{1}{2\pi\sqrt{LC}}$$



Variation of Current near Parallel Resonance

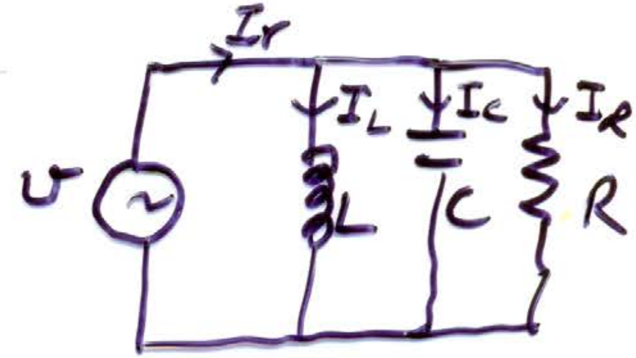


Variation of Susceptance, Admittance with frequency in Parallel ckt



Variation of Current with frequency in Parallel Resonant ckt

Q-FACTOR OF PARALLEL RESONANCE



The Condition of resonance occurs when susceptance is Zero

$$Y = G + jB$$

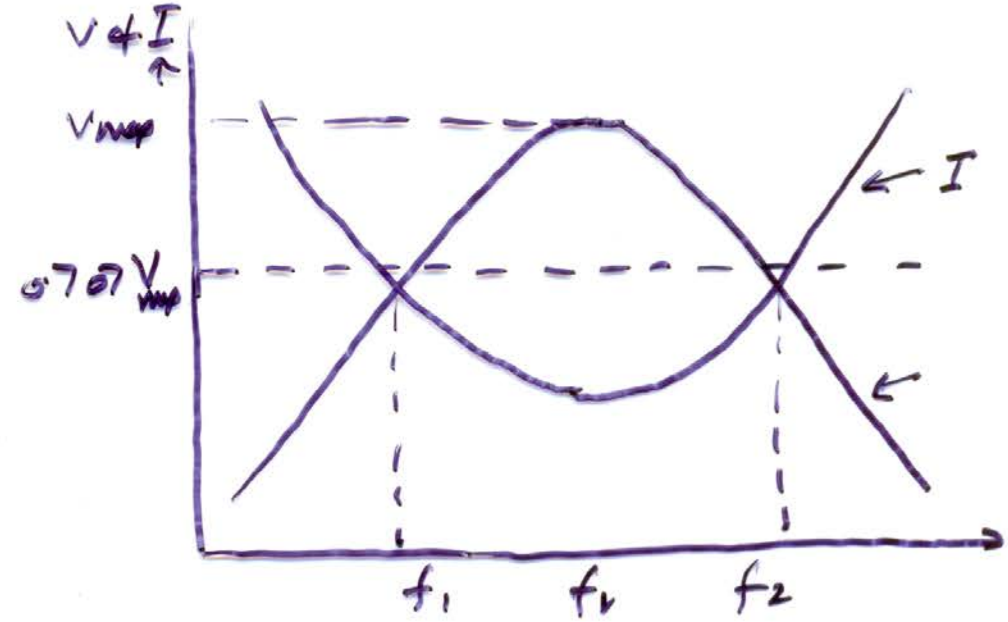
$$= \frac{1}{R} + j\omega C + \frac{1}{j\omega L}$$

$$= \frac{1}{R} + j\left(\omega C - \frac{1}{\omega L}\right)$$

$$\omega_r C = \frac{1}{\omega_r L}$$

$$\omega_r = \frac{1}{\sqrt{LC}}$$

Bandwidth $= f_2 - f_1$



<u>For parallel circuit</u>

$$\omega_1 C - \frac{1}{\omega_1 L} = -\frac{1}{R} \quad \text{---(I)}$$

$$\omega_2 C - \frac{1}{\omega_2 L} = \frac{1}{R} \quad \text{---(II)}$$

$$\omega_1^2 + \frac{\omega_1}{RC} - \frac{1}{LC} = 0$$

$$\omega_1 = -\frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}}$$

$$\omega_2 = \frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}}$$

$$\omega_2 - \omega_1 = \frac{1}{RC} = \frac{\omega_r}{R\,\omega_r C}$$

$$\boxed{BW = \frac{1}{RC}}$$

$$Q_r = \frac{\omega_r}{\omega_2 - \omega_1} = \frac{\omega_r}{1/RC} = \omega_r RC$$

$$Q_r = \frac{2\pi \times \frac{1}{2} L I^2}{\frac{I^2}{2} R \times \frac{1}{f}} = \frac{2\pi f \cdot \frac{1}{2} L \left(\frac{V}{\omega L}\right)^2 \cdot R}{\frac{V^2}{2}}$$

$$= \frac{2\pi f L R}{\omega^2 L^2} = \frac{R}{\omega L}$$

$$Q = 2\pi \frac{\frac{1}{2} C V^2}{\left(\frac{V^2}{2R}\right) \times \frac{1}{f}} = 2\pi f C R = \omega C R$$

Current Magnification occurs in parallel Resonant cct.

$$I_L = \frac{V}{\omega_r L} = \frac{IR}{\omega_r L} = I Q_r; \quad I_c = \frac{V}{1/\omega_r C} = \frac{IRC}{\omega_r} = I Q_r \qquad \underline{V = IR}$$

COMPARISON B/W SERIES & PARALLEL RESONANCE

S No	Point of Comparison	SERIES	PARALLEL
1.	Impedance	Minimum 	maximum 
2	Current	maximum 	minimum 
3.	$f < f_r$	$X_C > X_L$	$B_L > B_C$
4.	$f > f_r$	$X_L > X_C$	$B_C > B_L$
5	p.f	Unity	Unity
6	Q-factor	$Q = \frac{\omega L}{R}$	$Q = \frac{1}{R}\sqrt{\frac{L}{C}}$
7	f_r	$\frac{1}{2\pi\sqrt{LC}}$	$\frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$
8	Magnification	Voltage	Current
9	Z	$Z = R$	$Z = \frac{L}{CR}$
10	Admittance	maximum	minimum.
11	Type of behaviour	Selective (Acceptance cct)	Rejective

Parallel Resonance

Parallel Resonance

$$w_o = \frac{1}{\sqrt{LC}}$$

$$Q = \frac{w_o L}{R}$$

$$BW = (w_2 - w_1) = w_{BW} = \frac{R}{L}$$

$w_1 w w_2$

$$w_1, w_2 = \left[\frac{\mp R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}} \right]$$

$$w_1, w_2 = w_o \left[\frac{\mp 1}{2Q} + \sqrt{\left(\frac{1}{2Q}\right)^2 + 1} \right]$$

Series Resonance

$$w_o = \frac{1}{\sqrt{LC}}$$

$$Q = w_o RC$$

$$BW = w_{BW} = \frac{1}{RC}$$

$$w_1, w_2 = \left[\frac{\mp 1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}} \right]$$

$$w_1, w_2 = w_o \left[\frac{\mp 1}{2Q} + \sqrt{\left(\frac{1}{2Q}\right)^2 + 1} \right]$$